

# Action for particles faster than light

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## Abstract

A general action for particles faster than light is presented. It is demonstrated that this action is invariant under reparametrizations. For several cases, it is shown that in the high velocity regime the action is invariant under anisotropic space-time transformation and at quantum level the system has fractal behavior. For those cases, it is shown that the action describes a particle in Finsler geometry and equivalent to one dimensional field theory in a curved space, where the metric depends on temporal derivatives.

## 1 Introduction

Special relativity has been one of most important theories in physics. However, recently OPERA collaboration has reported particles faster than light [1]. Some authors think that we have to wait another experiment to confirm that result while others think that such experiment might has mistakes or that has not been well interpreted [2]. But, if that experiment is right,

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special relativity must be changed. There are several theories with breaking Lorentz symmetry. For example, P. Hořava has proposed a gravity theory with breaking Lorentz symmetry, which is renormalizable [3]. In High Energy Physics there are some extensions of the standard model that consider Lorentz symmetry violation [4, 5]. Also, it is worth to mention that MOND theory is an alternative propose to dark matter [6, 7] and breaks Lorentz symmetry too [8]. Many of these theories have important properties, for example, Hořava's gravity has an alternative mechanism to inflation [9] and can explain some cosmological phenomena without dark matter [10]. Doubly Special Relativity (DSR) is another interesting theory with breaking Lorentz symmetry [11, 12, 13]. Other works with breaking Lorentz symmetry can be seen in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

According to special relativity, the action for a particle with mass  $m$  and velocity  $u$  is

$$S = -m \int dt \sqrt{1 - u^2}, \quad (1)$$

where  $t$  is the time. In this paper we take light velocity  $c = 1$ .

If  $u$  is near from 1, we can take  $u = 1 + \delta$ , where  $|\delta| \ll 1$ . In this case we find  $\sqrt{1 - u^2} \approx \sqrt{-2\delta}$ . We can see that if  $\delta > 0$ , the particle is faster than light and the action  $S$  is nonsense, for example OPERA reported  $\delta = (2.37 \pm 0.32(stat.)_{-0.24}^{+0.34}(sys.)) \times 10^{-5}$ . However, if there are particles faster than light, this action must be only an approximation of another one that makes sense if  $u > 1$ . In that case we have to change

$$\sqrt{1 - u^2} \quad (2)$$

for another term. Notice that in this case, Lorentz transformation is only a limit of another symmetry.

In this work we present a general action for particles faster than light and demonstrate that it is invariant under reparametrizations. First we propose a simple model and show that in high velocity regime its action is invariant under anisotropic space-time transformation and at quantum level the system has fractal behavior. Then, it is possible that in the high energy regimen a particle has that kind of behavior. Interestingly, Hořava gravity has fractal

properties [33] and other authors argued that at Plank scale the space-time has fractal properties as well [34, 35, 36]. Also, we show that these kind of actions are equivalent to one dimensional field theory in a curved space, where the metric depends on temporal derivatives. Moreover we show that there is a relation between those actions and Finsler geometry. It is worth mentioning that recently Finsler geometry was proposed as an generalized Minkowski geometry [37, 38, 39].

This work is organized in the following way: In section 2 the general action is presented; in section 3 a simple model is studied; in section 4 the relation between our model and Finsler geometry is analyzed and finally in section 5 a brief summary is given.

## 2 Action

In this section we propose a new action that describes particles faster than light and reduces to the usual relativistic one.

The action for a relativistic particle is

$$S = -m \int d\tau \sqrt{\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{d\vec{x}}{d\tau}\right)^2}, \quad \dot{t} = \frac{dt}{d\tau}, \quad (3)$$

which is invariant under repametrizations in  $\tau$ :

$$\tau \rightarrow \tau = \tau(\omega). \quad (4)$$

Notice that if  $\tau = t$  we obtain (1).

Let  $f$  be a function that if  $u \leq 1$ , then  $f(u^2) \approx \sqrt{1 - u^2}$ . But if  $u > 1$ , it makes sense. Thus, the action

$$S = -m \int d\tau L, \quad L = \frac{dt}{d\tau} f(u^2), \quad u^2 = \left(\frac{d\vec{x}}{d\tau} \frac{1}{\frac{dt}{d\tau}}\right)^2 \quad (5)$$

is a generalization of (3). We can see that (5) is also invariant under reparametrizations in  $\tau$ . Now, using (5) we find

$$E = -P_t = -\frac{\partial L}{\partial \dot{t}} = m \left( f(u^2) - 2u^2 \frac{df(u^2)}{du^2} \right), \quad (6)$$

$$P_i = \frac{\partial L}{\partial \dot{x}^i} = -2m \frac{df(u^2)}{du^2} \frac{\dot{x}_i}{\dot{t}}, \quad P = 2mu \left| \frac{df(u^2)}{du^2} \right|, \quad (7)$$

then

$$H = P_t \dot{t} + \vec{P} \cdot \frac{d\vec{x}}{d\tau} - L = 0, \quad (8)$$

and

$$\frac{E}{P} = \left[ \frac{f(u^2)}{2u \left| \frac{df(u^2)}{du^2} \right|} - \left( \frac{\frac{df(u^2)}{du^2}}{\left| \frac{df(u^2)}{du^2} \right|} \right) u \right]. \quad (9)$$

When this equation is invertible, we can express  $u$  as a function of  $E$  and  $P$ , namely  $u = u(E, P)$ . Then, using (6) and (7) we have a constraint

$$\phi(E, P) = 0. \quad (10)$$

Therefore, using the Dirac's method [41], the extended Hamiltonian is given by

$$H_{ext} = \lambda \phi(E, P), \quad (11)$$

where  $\lambda$  is a Lagrange multiplier. Then, the Hamiltonian action is

$$S_H = \int d\tau \left( P_t \dot{t} + P_i \dot{x}_i - \lambda \phi(P_t, P_i) \right). \quad (12)$$

## 2.1 Alternative actions

The action (5) has at least three alternative actions. First, we consider

$$S_I = -\frac{1}{2} \int d\tau \dot{t} \left[ \frac{(f(u^2))^2}{\lambda} + \lambda m^2 \right]. \quad (13)$$

The equation of motion for  $\lambda$  implies

$$\lambda = \frac{f(u^2)}{m}, \quad (14)$$

substituting this result in (13) we obtain (5). We can see that, unlike (13), in this action the case  $m = 0$  makes sense.

Another alternative action is

$$S_{II} = -m \int d\tau \dot{t} \left[ f(\lambda) + (u^2 - \lambda) \frac{df(\lambda)}{d\lambda} \right]. \quad (15)$$

For this, the equation of motion for  $\lambda$  gives

$$\lambda = u^2, \quad (16)$$

substituting this result in (15) we obtain (5).

Now, let us consider

$$S_{III} = -\frac{1}{2} \int d\tau \dot{t} \left[ \frac{(f(\beta))^2 + (u^2 - \beta) \frac{d(f(\beta))^2}{d\beta}}{\lambda} + \lambda m^2 \right]. \quad (17)$$

Using the equations of motion for  $\lambda$  and  $\beta$ , we get (5).

### 3 Anisotropic cases

A simple model which describes particles faster than light is

$$S = -m \int d\tau \frac{dt}{d\tau} \sqrt{1 - u^2 + \alpha u^{2n}}. \quad (18)$$

Notice that, when  $u \gg 1$ , we obtain

$$S \approx -m\sqrt{\alpha} \int d\tau \frac{(\dot{x}^2)^{\frac{n}{2}}}{t^{n-1}}. \quad (19)$$

This action is invariant under anisotropic transformations

$$t \rightarrow b^z t, \quad \vec{x} \rightarrow b\vec{x}, \quad z = \frac{n}{n-1}. \quad (20)$$

Therefore, we get

$$P_t = m\sqrt{\alpha}(n-1) \frac{(\dot{x}^2)^{\frac{n}{2}}}{t^n}, \quad (21)$$

$$P_i = -m\sqrt{\alpha} \frac{(\dot{x})^{n-2}}{t^{n-1}} \dot{x}_i \quad (22)$$

and the dispersion relation

$$E^2 = \frac{(n-1)^2}{(m^2\alpha)^{\frac{1}{n-1}}} (P^2)^{\frac{n}{n-1}}. \quad (23)$$

Then, in the quantum theory the wave equation is given by

$$-\frac{\partial^2\psi}{\partial t^2} = \frac{(n-1)^2}{(m^2\alpha)^{\frac{1}{n-1}}} (-\nabla^2)^{\frac{n}{n-1}} \psi, \quad \hbar = 1. \quad (24)$$

It is a fractional wave equation [42]. Fractional-like wave equations describe systems with fractal properties [43, 44]. Then, it is possible that in the high energy regimen a particle has fractal properties. It is worth to mention that Hořava gravity has similar properties [33] and other authors argued that at Plank scale the space-time has fractal properties as well [34, 35].

## 4 Relation with Finsler geometry

Recently Finsler geometry was proposed as an alternative to the Minkowski one in the regime close to the Plank scale [37, 38, 39, 40]. This geometry has been proposed as brackground for some theories, like MOND [45], very special relativity [46] and DSR [47]. In this geometry the action is

$$S = \int d\tau F(x^\mu, \dot{x}^\mu), \quad (25)$$

where  $F(x^\mu, \dot{x}^\mu)$  is a homogeneous function of one degree of  $\dot{x}^\mu$ . For example, if  $\gamma_{\mu\nu}(x^\mu, \dot{x}^\mu)$  is a homogeneous function of zero degree of  $\dot{x}^\mu$ , then

$$S = \int d\tau \sqrt{\gamma_{\mu\nu}(x, \dot{x}) \dot{x}^\mu \dot{x}^\nu} \quad (26)$$

is Finsler-like action.

In this section we show a relation between action (18) and the Finsler one (26). Let us notice that the action (18) can be written as

$$S = -m \int d\tau \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \quad (27)$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + Z_{\mu\nu}, \quad Z_{\mu\nu} = \begin{pmatrix} 0 & \cdots \\ \vdots & \alpha \left( \frac{\dot{x}^2}{t^2} \right)^{n-1} \delta_{ij} \end{pmatrix}. \quad (28)$$

This metric depends on velocity and if  $\dot{x}^\mu \rightarrow \Omega \dot{x}^\mu$ , then  $g_{\mu\nu} \rightarrow g_{\mu\nu}$ . Therefore (18) is a Finsler-like action.

In general, if  $h(u^2)$  is a regular function, the action

$$S = -m \int d\tau \sqrt{1 - u^2 + h(u^2)} \quad (29)$$

can be expressed like

$$S = -m \int d\tau \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \quad (30)$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + Z_{\mu\nu}, \quad Z_{\mu\nu} = \begin{pmatrix} 0 & \cdots \\ \vdots & \frac{h(u^2)}{u^2} \delta_{ij} \end{pmatrix}. \quad (31)$$

Also, this is Finsler-like action. Notice that, for several cases, the general action (5) is Finsler-like action too. Then it is possible that for particles faster than light Finsler geometry replaces the Minkowski geometry.

Additionally, the action (30) can be written as

$$S = \int d\tau \sqrt{G} (G^{\tau\tau} g_{\mu\nu} \partial_\tau x^\mu \partial_\tau x^\nu + \Lambda), \quad G_{\tau\tau} = \lambda^2, \quad \Lambda = m^2 \quad (32)$$

This action can be interpreted as a 1-dimensional field in a curve space with metric  $G_{\tau\tau}$ , cosmological constant  $\Lambda$  and background  $g_{\mu\nu}$ .

## 5 Summary

In this work we presented a general action for particles faster than light. We demonstrated that this action is invariant under reparametrizations. For several cases, it was shown that in the high velocity regime the action is

invariant under anisotropic space-time transformation and at quantum level the system has fractal behavior. It is worth to mention that other authors have proposed theories, like Hořava gravity, with fractal properties. Moreover, for the same cases, it was shown that the action describes a particle in Finsler geometry and equivalent to one dimensional field theory in a curved space, where the metric depends on temporal derivatives.

Then, if there are particles faster than light, we would ample ground for further research in spacetime-symmetry physics. However, we have to wait more experiments to find new physics, in particular to know the action for a free particle.

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